

# Grand unification for mirror fermions

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## Abstract

The possibility of grand unification of the standard model (SM) with fermion spectrum extended to include mirror fermions is examined. SM gauge couplings do not automatically unify.  $SO(10)$  grand unification is studied with one intermediate scale. Renormalization group equations (RGE) for fermion Yukawa couplings and the scalar self-coupling are studied numerically at one and two loop level. Strong restrictions for mirror fermion masses are obtained assuming perturbative unification. Mirror masses much smaller than the tree unitarity bounds are required. In particular mirror leptons have to be around 50 GeV. Consistency of the mirror fermion model with LEP precision data is established. A direct search for single production of mirror neutrinos at LEP could exclude or confirm the GUT version of the mirror fermion model.

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# 1 Introduction

In this note we study the possibilities of perturbative grand unification for the mirror fermion extension of the SM. This very simple extension of the SM [1] is to enlarge only the fermion content by introducing mirror (i.e. opposite chirality property) fermions to each fermion of the SM (i.e. to each ordinary fermion), preserving the  $SU(3) \otimes SU(2) \otimes U(1)$  group structure. Ordinary and mirror fermions are allowed to mix. In fact mixing is necessary in order to avoid stable mirror fermions. Present experiments directly exclude mirror fermions with masses below roughly half of the  $Z^0$  mass. Heavier mirror fermions are still allowed. Many of the phenomenological consequences of such a model have been worked out in [2], [3], and motivations were summarized in [4]. (For a review of earlier work on models with mirror fermions see [5].)

The mixing angles of ordinary and mirror fermions are small. Constraints from experimental data have been worked out in [6]. Mixing angles are typically bounded by 0.1 - 0.2. Even more restrictive upper bounds (0.02) for the leptonic mixing angles were obtained in [7]. Also a recent fit to LEP precision data shows that the model is consistent with experiment for small (but non-zero) mixing angles [8]. Since ordinary (left or right) and mirror fermions (right or left, respectively) transform identically under the  $SU(3) \otimes SU(2) \otimes U(1)$  group it is possible to write down invariant mixing mass terms. Such mass terms of the order of the symmetry breaking scale would imply large mixings, therefore small mixing angles are imposed as an experimental constraint. In GUT it is easy to forbid the mixing mass terms invoking discrete symmetries, therefore in the following we neglect mixing effects.

We emphasize that we are concerned with the unification of the above described simple mirror fermion model assuming minimal extension of the standard model. Thus in our case mirror fermions get a mass at the scale of electroweak symmetry breaking.

## 2 Gauge coupling unification

It is well known that starting with the measured values of the gauge couplings, the running couplings do not meet at a single scale [9] in the SM. So it is interesting to examine whether mirror fermions change the situation. Since (by assumption) each ordinary fermion has a mirror partner of similar quantum numbers, it is easy to see that at one loop the slope of the running couplings gets the same contribution for all the three gauge couplings. Thus only the actual values of the couplings are modified, but they do not meet, similarly to the SM case. We have checked that neither threshold effects nor two loop RG effects change this conclusion.

Clearly, we may have GUT only with symmetry breaking in at least two steps. We have chosen  $SO(10)$  as the grand unifying group. Among other attractive features [10] we want to keep the number of new fermions small. Assuming two step symmetry breaking the intermediate scale as well as the GUT scale is determined by the two loop RG equations and low energy experimental input couplings [11]. Using the notation of [11] we have

$$\frac{d\omega_i(\mu)}{d \ln \mu} = -\frac{a_i}{2\pi} - \sum_j \frac{b_{ij}}{8\pi^2 \omega_j}, \quad (1)$$

where

$$\omega_i = \alpha_i^{-1} = 4\pi/g_i^2. \quad (2)$$

The initial values are:  $\alpha_1(M_Z) = 0.016887 \pm 0.000040$ ,  $\alpha_2(M_Z) = 0.03322 \pm 0.00025$ ,  $\alpha_3(M_Z) = 0.120 \pm 0.008$ . Between  $M_Z$  and  $M_I$  (the intermediate scale) the constants are given by

$$a = \begin{pmatrix} \frac{81}{10} \\ -\frac{5}{6} \\ -3 \end{pmatrix}, \quad b = \begin{pmatrix} \frac{1149}{50} & \frac{9}{2} & \frac{264}{15} \\ \frac{3}{2} & \frac{329}{6} & 24 \\ \frac{11}{5} & 9 & -20 \end{pmatrix}. \quad (3)$$

Between  $M_I$  and  $M_U$  (unification scale) the constants  $a_i$  and  $b_{ij}$  depend on the intermediate range unbroken gauge group  $G_I$  as well as the Higgs multiplets remaining massless at the scale  $M_I$ . We consider all the possibilities listed in [11] i.e.  $G_I = \{2_L 2_R 4_C\}$ ,  $\{2_L 2_R 4_C \otimes P\}$ ,  $\{2_L 2_R 1_X 3_c\}$ ,  $\{2_L 2_R 1_X 3_c \otimes P\}$ ,  $\{2_L 1_R 4_C\}$ ,  $\{2_L 1_R 1_X 3_c\}$ .  $\{2_L 2_R 4_C\}$  e.g. stands here for the group  $SU(2)_L \otimes SU(2)_R \otimes SU(4)_C$ .  $X = (B - L)/2$  and the factor  $P$  is an unbroken parity symmetry. The Higgs content does not change by considering the mirror fermion model so it is the same as given in Table I of [11]. The  $a_i$ 's all increase by 4 as compared to [11], the change of the  $b_{ij}$ 's is more complicated. To save space we do not reproduce the actual values here.

Solving the RG equations and applying the appropriate (2-loop) matching conditions we arrive at the scales and couplings at  $M_U$  as listed in Table I. There is no solution for  $G_I = \{2_L 1_R 1_X 3_c\}$ . Not all these solutions are acceptable, because of the constraint coming from the experimental lower limit on the proton lifetime. Chains 1a, 2a survive, 1b and 2b are marginal, while chain 3 is definitely ruled out.

## 3 Scalar couplings

### 3.1 RG equations for the Yukawa couplings

As well known the running of Yukawa couplings suffers from Landau poles in the one loop approximation. The presence of these singularities at some scale  $\Lambda_{Yuk.}$  highlights the breakdown of perturbation theory and the probable triviality of the continuum limit. It is reasonable to accept as perturbative regions those scales, where the squared Yukawa couplings are less than  $4\pi$ , (i.e.  $\alpha_{Yuk.} \leq 1$ .) The RG equations for the Yukawa couplings are given in a general gauge theory in [12] for both the one and two loop case. The initial values are given by relating fermion masses to the Yukawa couplings at threshold. Even though the fermion mass spectrum is not known (top and mirror fermion masses are unknown), we start the Yukawa coupling evolutions from thresholds assuming 'reasonable' masses. Useful guides on the mirror fermion masses are the experimental lower bounds and also the tree unitarity upper bounds derived in [3]. Mirror doublets are always assumed to be degenerate as required to reproduce the precision LEP data ([8]). Moreover it is natural to assume that mirror fermions are always heavier than the corresponding ordinary fermions (this is non trivial for the mirror top only.) For the top quark mass we assume values consistent with the fit of precision LEP data. The other condition on the running Yukawa couplings is that  $\alpha_{Yuk.}$  should remain less than one below  $M_I$ , (i.e.  $\Lambda_{Yuk.} \geq M_I$ ). The latter is determined from the gauge coupling RG equations assuming one of the possible two step symmetry breakings of  $SO(10)$  as explained above.

We have solved the one loop RG equations numerically for many representative choices of masses. We found that mirror leptons should be light, much lighter than mirror quarks in order to get 'reasonably' high masses at all. This conclusion is in accord with [13], where Yukawa coupling evolution is studied starting from a high scale. Namely, the (mathematical) infrared fixed point is reached at vanishing lepton masses. Also the mirror quark masses should be relatively small. This is again consistent with [13], where an upper bound on the sum of quark mass squares is derived. Some numerical examples are:  $M_{mirror} = 92$  GeV,  $M_{top} = 150$  GeV yields  $\Lambda_{Yuk.} = 10^{10.26}$  GeV;  $M_{m.lepton} = 50$  GeV,  $M_{m.quark} = 92$  GeV,  $M_{top} = 150$  GeV yields  $\Lambda_{Yuk.} = 10^{22.03}$  GeV;  $M_{m.lepton} = 50$  GeV,  $M_{m.quark} = 92$  GeV,  $M_{m.top} = M_{m.bottom} = M_{top} = 145$  GeV yields  $\Lambda_{Yuk.} = 10^{11.1}$  GeV. Compared to the tree unitarity upper bounds of [3] these values of mirror fermion masses are rather small. In particular in [8] also higher masses in the range (100-300) GeV have been assumed.

### 3.2 RG equation for the scalar quartic self coupling

Already at one loop we have a coupled system of differential equations for the gauge, Yukawa and quartic couplings. The RG equations are given in the general case for one and two loop level in [12] and [14]. Though the Higgs mass is unknown, we continue our practice to assume a 'reasonable' value to provide an initial condition to the RG equation. Following [15] the RG evolution of the scalar quartic coupling may be used to establish upper and lower bounds on the Higgs mass. The lower bound arises from requiring a positive quartic coupling ( $\lambda$ ). The upper bound arises from the Landau pole of  $\lambda$  (triviality bound). The corresponding scales are:  $\Lambda_{inst.}$  and  $\Lambda_\lambda$ . In general  $\Lambda_{inst.} \leq \Lambda_\lambda \leq \Lambda_{Yuk.}$  for given fermion masses. Combining these with the information obtained from gauge coupling evolution for  $M_I$ , we have the condition  $M_I \leq \Lambda_{inst.}$ , i.e. only sufficiently high  $\Lambda_{inst.}$  is acceptable.

Solving the RG equations numerically we find that depending on the choice of fermion masses the allowed range of the Higgs mass is rather restricted or even empty. Nevertheless for reasonably light mirror leptons and mirror quarks the perturbative region may extend to  $M_X$ . Some numerical examples are: for  $M_{mirror} = 92$  GeV,  $M_{top} = 150$  GeV, ( $\Lambda_{Yuk.} = 10^{10.26}$  GeV),  $M_{Higgs} \in (222.6 \text{ GeV}, 222.9 \text{ GeV})$  is acceptable for chain 1a and  $M_{Higgs} \in (221.9 \text{ GeV}, 225.5 \text{ GeV})$  is acceptable for chain 2a. For  $M_{m.lepton} = 50$  GeV,  $M_{m.quark} = 92$  GeV,  $M_{top} = 150$  GeV, ( $\Lambda_{Yuk.} = 10^{22.03}$  GeV)  $M_{Higgs} \in (193 \text{ GeV}, 227 \text{ GeV})$  is acceptable for chain 1a and  $M_{Higgs} \in (190 \text{ GeV}, 238 \text{ GeV})$  for chain 2a. For  $M_{m.lepton} = 50$  GeV,  $M_{m.quark} = 92$  GeV,  $M_{m.top} = M_{m.bottom} = M_{top} = 145$  GeV ( $\Lambda_{Yuk.} = 10^{11.1}$  GeV)  $M_{Higgs} \in (240.63 \text{ GeV}, 242.5 \text{ GeV})$  is acceptable for chain 1a and  $M_{Higgs} \in (239.8 \text{ GeV}, 246 \text{ GeV})$  for chain 2a.

### 3.3 Two loop RG effects for scalar couplings

Using the complete two loop RG equations, besides the infrared fixed point, the possibility of a second (ultraviolet) fixed point arises. Instead of trying to solve the nonlinear equations determining the second fixed point, we have solved the RG equations numerically and observed the peculiar scale dependence of the couplings. Namely, below the second fixed point the scalar couplings are almost constant (this corresponds to the infrared fixed point) and after a short transition period the couplings are again constant at different values. The transition scale is near the Landau pole of the one loop RG equations. The other possibility is that new ultraviolet fixed points do not arise, so the Landau pole does not disappear. An important question to

answer is, whether or not the fixed point behaviour belongs to the perturbative range or not, i.e. do couplings remain sufficiently small (so that e.g. the 'fine structure constants' associated to the different couplings are all less than unity.) We find that this does not happen. The perturbative regime does not appreciably change when the one loop RG equations are replaced by the two loop ones. It follows that the above one loop results on the allowed range of mirror fermion and Higgs masses will not be changed by the more sophisticated two loop treatment, provided that perturbative unification is assumed. An example of a simplified model is shown in figs. 1,2. We have kept only the top and mirror quarks, the Higgs and the QCD coupling. The masses are  $M_{top} = 150$  GeV,  $M_{m.quark} = 92$  GeV and  $M_{Higgs} = 300$  GeV.

## 4 Constraints implied by LEP data

### 4.1 Precision data

At low energies the really crucial test of any theory beyond the SM is whether it survives a comparison with LEP precision data. For the mirror fermion model such a comparison has been performed in [8]. Since that paper assumed somewhat higher mirror masses than allowed by perturbative unification the analyses has to be repeated with lower mirror masses. The fitted parameters are quark mixing angles ( $\alpha_q$ ) assumed to be equal for u, c quarks (those of d and s quarks are not free parameters) and the top quark mixing angle ( $\alpha_{top}$ ) (the bottom quark mixing angle is not a free parameter). We have used preliminary 1992 data as given in [16]. In [8] it was found that zero mixing angles are already excluded, but for suitable mixing angles very good fits to LEP and low energy neutrino data are obtained. This qualitative statement remains unchanged for the lower mirror masses as well. An example of the equal  $\chi^2$  curves is given in fig. 3 for the following input parameters:  $M_{m.lepton} = 50$  GeV,  $M_{m.quark} = 92$  GeV,  $M_{top} = M_{m.top} = M_{m.bottom} = 150$  GeV,  $M_{Higgs} = 250$  GeV,  $\alpha_s = 0.12$  and the right leptonic mixing angles are zero, the left leptonic mixing angles are equal to 0.092. Good fits for lower values of the left leptonic mixing angles are possible at the expense of increasing the top mass.

### 4.2 Direct search for single production

The pair production of mirror fermions is excluded by experimental data. However, single production through the mixing vertex coupling ordinary and mirror fermions to the weak vector bosons is also possible. (The same vertex is responsible for the decay, which goes into an ordinary fermion and a possibly virtual vector boson. Decay to ordinary lepton and a photon occurs only in second order and is very small.) The cross-section depends on the mixing angles (it is given e.g. in [17]). Therefore experiment will give combined upper limits on mirror fermion masses and mixings. To our knowledge such a search has not been performed so far. The L3 search for singly produced excited neutrinos decaying to  $eW$  [18] can be used to derive a rough upper bound on the mixing angles as a function of mirror neutrino mass. (The estimate is very rough, since the angular distribution of mirror neutrino and excited neutrino is very different.) For  $M_{m.lepton} = 50$  GeV, zero right leptonic mixing angles and equal left leptonic mixing angles, we got an upper bound of 0.054 for the latter. Thus, a systematic search of single production of mirror neutrinos combined with other LEP precision data may easily exclude the low masses required by the GUT scenario.

## 5 Conclusion

Assuming perturbative grand unification for the mirror fermion extension of the standard model, we find that intermediate scale symmetry breaking is necessary. Assuming two step symmetry breaking the intermediate scale can be determined from gauge coupling evolution. Mirror fermion masses are severely restricted by the requirement that Yukawa couplings should remain small during the evolution below the intermediate scale. Quite restrictive information on the Higgs mass is obtained assuming positive and small quartic coupling during evolution. Mirror lepton masses turn out to be small (near half of the  $Z$  mass), mirror quark masses should be also much smaller than allowed by the tree unitarity bounds. For suitable mixing angles LEP precision data can be very well fitted with the mirror masses allowed by the above considerations. A combined study of LEP precision data with a direct search for single mirror neutrino production could easily exclude the low mirror lepton masses required by perturbative grand unification.

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## Table caption

**Table 1.** Intermediate scale ( $M_I$ ) and unification scale ( $M_U$ ) obtained by solving the renormalization group equations for different intermediate symmetry groups ( $G_I$ ).

**Table 1**

Chain	$G_I$	$\log_{10}(M/1 \text{ GeV})$		$\omega_U$
		$M_I$	$M_U$	
1a	$2_L 2_R 4_C$	10.11	16.32	23.58
1b	$2_L 2_R 4_C \otimes P$	13.70	14.85	21.01
2a	$2_L 2_R 1_X 3_c$	9.35	16.37	23.67
2b	$2_L 2_R 1_X 3_c \otimes P$	10.66	15.34	22.94
3	$2_L 1_R 4_C$	11.30	14.40	24.87

## Figure captions

**Fig. 1.** The running couplings as a function of  $t = \log_{10}(\Lambda)$  at one loop order in the simplified model containing only the top quark and mirror quarks, Higgs and the QCD coupling. The input masses are  $M_{top} = 150$  GeV,  $M_{m.quark} = 92$  GeV and  $M_{Higgs} = 300$  GeV.

**Fig. 2.** The same as fig. 1 at two loop order.

**Fig. 3.** Equal  $\chi^2$  curves of a fit to LEP and low energy data. The area between the indicated curves belongs to the lowest  $\chi^2$ . The other curves correspond to  $\chi^2$ 's increasing by steps of 0.5.

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